A NOVEL LOSSY MICROSTRIP DISPERSION MODEL

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ABSTRACT

A new model is presented for evaluating the microstrip dispersion. The proposed model is simple and is based on the two-dimensional field equations of the microwave planar circuits. Quantitative results obtained fit well available experimental data and are compared with those corresponding to other models for both lossless and lossy structures.

INTRODUCTION

In this paper a new model for microstrip lines is presented, based on a rigorous electromagnetic theory for quasistationary fields [1]. This model was first constructed for the case of the lossless microstrips [2] and it is expected to have a larger range of validity for evaluating the microstrip dispersion, as compared to existing models [3]-[8].

The first order two-dimensional equations derived from the quasistationary field theory of the microwave planar structures [1]-[2] for time-harmonic conditions can be written in the form

$$-\nabla U = Z_s \mathbf{J}_s$$

$$-\nabla \cdot \mathbf{J}_s = Y_s U \tag{1}$$

where U is the transverse voltage between the strip and the conducting base, $\mathbf{J}_s$ is the surface current density carried by the conducting strip, $Z_s = R_s + j\omega L_s$ is the longitudinal impedance per square , and $Y_s = G_s + j\omega C_s$ is the transverse admittance per unit area. The corresponding second order equations are

$$\nabla^2 U - \gamma^2 U = 0$$

$$\nabla^2 \mathbf{J}_s - \gamma^2 \mathbf{J}_s = 0 \tag{2}$$

where $\gamma^2 = Z_s Y_s$.
These equations are to be used in the computation of the microstrip dispersion on the basis of the proposed model.

MODEL CONSTRUCTION

The effective dielectric constant for a given microstrip structure is a function of frequency and can be expressed as

$$\epsilon_{eff}(\omega) = f\left(Z_0, \frac{w}{h}, \epsilon_r\right) \tag{3}$$

where Z_0 is the characteristic impedance, h is the height of the substrate, w is the width of the stripline, and ϵ_r is the relative permittivity of the substrate. The dispersion of a lossless microstrip can be calculated by using a simple model in which the static parameters per unit length of the real structure, L' and C', are kept unchanged. For the microstrip shown in Fig. 1, the model contains a parallel-plate capacitor, completed with lateral walls of $\epsilon = 0$ and $\mu \to \infty$, such that the field exists only within the region between the plates. The model is presented in Fig. 2, with a dielectric between the plates of same permittivity in the central part $y \in (-a, a)$ as in the original microstrip, a and b having to be determined from the condition that the corresponding propagation parameters at low frequency, $Z_0 = \sqrt{L'/C'}$ and the velocity $1/\sqrt{L'C'}$, be the same as in the original microstrip.

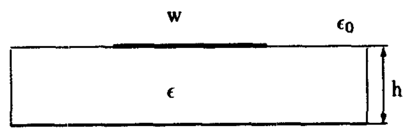

Fig. 1. Microstrip geometry.

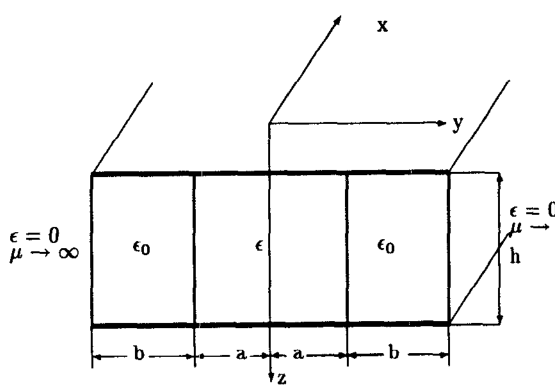

Fig. 2. The model.

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The idea is to apply this model at higher frequencies and to analyze its accuracy and its range of validity. Assuming the substrate to be nonmagnetic, we have to impose only the condition that the corresponding C' remains unchanged,

$$C' = \frac{\epsilon 2a}{h} + \frac{\epsilon_0 2b}{h} = \epsilon_{eff}(0)C_0 \quad (4)$$

with the capacitance of the structure filled with air

$$C_0 = \frac{2(a+b)\epsilon_0}{h} \quad (5)$$

With C_0 and $\epsilon_{eff}(0)$ assumed to be known, a and b can easily be determined in terms of ϵ_0, h, C_0, ϵ_r, and $\epsilon_{eff}(0)$.

CALCULATION OF MICROSTRIP DISPERSION

For practical microstrips $R \ll \omega L$ and $G \ll \omega C$, therefore in the estimation of the modal dispersion one can neglect the losses at a first approximation. In order to evaluate the attenuation α and the phase shift β_l for lossy microstrips, the following simpler algorithm is proposed: first, the effective dielectric constant is computed for the lossless structure (with R and G taken to be zero), which gives the dependence of the capacitance on frequency, and then the quantities α and β_l are determined from the expression corresponding to the classical (one-dimensional) theory of the lossy transmission lines, with the capacitance per unit length depending now on frequency.

Assume the line to be infinitely extended in the x-axis direction and choose the system of coordinates as shown in Fig. 2. Denoting $U_1(x,y)$ the voltage in the region $y \in (-a,a)$ and $U_2(x,y)$ the voltage in the lateral regions, $y \in (-(a+b),-b)$ and $y \in (b,a+b)$, the corresponding two-dimensional second order equations are

$$\frac{\partial^2 U_1}{\partial x^2} + \frac{\partial^2 U_1}{\partial y^2} + \gamma^2 U_1 = 0 \quad (6)$$

$$\frac{\partial^2 U_2}{\partial x^2} + \frac{\partial^2 U_2}{\partial y^2} + \beta_0^2 U_2 = 0 \quad (7)$$

where $\gamma^2 = \omega^2\epsilon\mu_0$ and $\beta_0^2 = \omega^2\epsilon_0\mu_0$. Assuming only the dominant mode of propagation, we look for solutions of the form

$$U_{1,2}(x,y) = Y_{1,2}(y)\exp(\pm j\beta x) \quad (8)$$

where β is real for lossless lines. From (6) and (7) we get

$$Y_1'' + (\gamma^2 - \beta^2)Y_1 = 0$$

$$Y_2'' + (\beta_0^2 - \beta^2)Y_2 = 0 \quad (9)$$

Since $\gamma^2 > \beta^2 > \beta_0^2$, we have $\alpha_1^2 \equiv \gamma^2 - \beta^2 > 0$ and $-\alpha_2^2 \equiv \beta_0^2 - \beta^2 < 0$. Equations (9) yield

$$Y_1 = A_1 \cos\alpha_1 y + B_1 \sin\alpha_1 y$$

$$Y_2 = A_2 \cosh\alpha_2 y + B_2 \sinh\alpha_2 y \quad (10)$$

Imposing the boundary conditions

$$U_1(x,a) = U_2(x,a) \quad (11)$$

$$J_{y1}(x,a) = J_{y2}(x,a) \quad (12)$$

$$J_{y2}(x,a+b) = J_{y2}(x,-a-b) = 0 \quad (13)$$

the following transcendental equation is derived

$$\xi \tan\xi - \sqrt{k^2-\xi^2}\tanh(m\sqrt{k^2-\xi^2}) = 0 \quad (14)$$

where $\xi \equiv \alpha_1 a$, $k^2 \equiv \omega^2\epsilon_0\mu_0(\epsilon_r - 1)a^2$, and $m \equiv b/a$. The dispersion [6] of the lossless microstrip is obtained as

$$\epsilon_{eff}(\omega) \equiv \frac{\beta^2}{\beta_0^2} = \epsilon_r - \frac{\xi^2}{k^2}(\epsilon_r - 1) \quad (15)$$

with the first root of the equation (14) corresponding to the dominant mode of propagation.

At very low frequencies, $\tan\xi \approx \xi$, $\tanh(m\sqrt{k^2-\xi^2}) \approx m\sqrt{k^2-\xi^2}$, and (14) and (15) yield

$$\epsilon_{eff}(0) = \epsilon_r - \frac{m}{m+1}(\epsilon_r - 1) \quad (16)$$

At very high frequencies, $k^2 \to \infty$, and we check that

$$\epsilon_{eff}(\infty) = \epsilon_r \quad (17)$$

Finally α and β_l for lossy microstrips are calculated in terms of their parameters per unit length, from

$$\alpha^2 = \frac{1}{2}\left\{\left[(R^2+\omega^2L^2)(G^2+\omega^2C^2)\right]^{\frac{1}{2}} + RG - \omega^2 LC\right\} \quad (18)$$

$$\beta_l^2 = \frac{1}{2}\left\{\left[(R^2+\omega^2L^2)(G^2+\omega^2C^2)\right]^{\frac{1}{2}} - RG + \omega^2 LC\right\} \quad (19)$$

where $C = \epsilon_{eff}(\omega)C_0$, $L = L' + \Delta L'$, R is given in [9], and $G = \omega C \tan\delta$, with $\tan\delta$ being the loss tangent of the dielectric and $\Delta L' = R/\omega$ the additional inductance due to the skin effect. The dispersion for the lossy microstrip is

$$\epsilon'_{eff}(\omega) = \frac{\beta_l^2}{\beta_0^2} \quad (20)$$

COMPARISON WITH EXPERIMENTAL RESULTS AND CONCLUSION

In Figs. 3-10 results obtained by using the model presented for both lossless and lossy microstrips are compared with those given by existing models and experimental data. The microstrip characteristics in Fig. 10 are: $w = 4.55 \times 10^{-3}$ m, $Z_0 = 29\ \Omega$, $\epsilon_r = 10.5$, $\tan\delta = 15 \times 10^{-4}$, and $\epsilon_{eff}(0) = 7.46$. For the calculation of the effective dielectric constant at zero frequency the formula in [10] was used. The characteristic impedance was calculated as in [11]. Various types of substrate have been considered, with ratios w/h both greater and smaller then unity. In spite of its simplicity, the proposed model yields a good agreement with the experimental data in all ranges of parameters considered, giving closer results then those in [7] for high frequencies.

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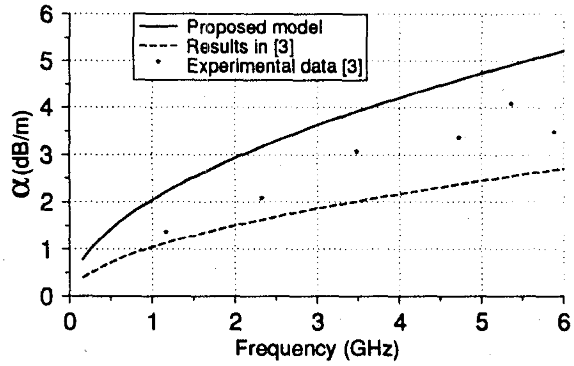

Fig. 3. Attenuation versus frequency for an alumina substrate and w/h=0.4.

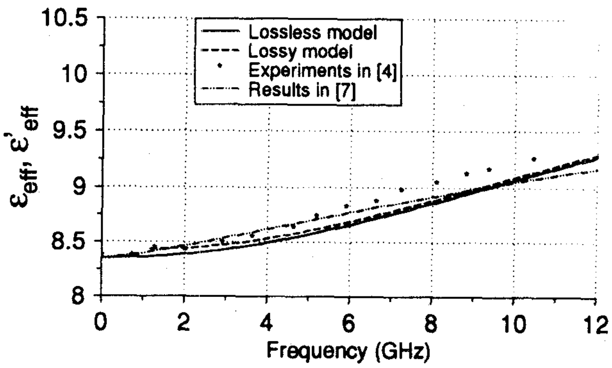

Fig. 4. Effective dielectric constant versus frequency for an alumina substrate with $\epsilon_r = 10.51$ and w/h=5.

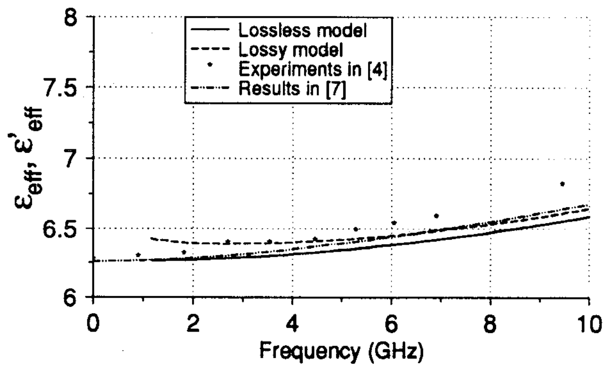

Fig. 5. Effective dielectric constant versus frequency for an alumina substrate with $\epsilon_r = 10.185$ and w/h=0.2.

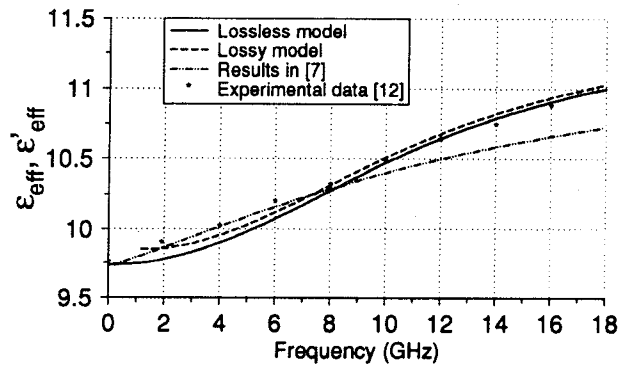

Fig. 8. Effective dielectric constant versus frequency for a sapphire substrate with $\epsilon_r = 11.5$ and w/h=9.1.

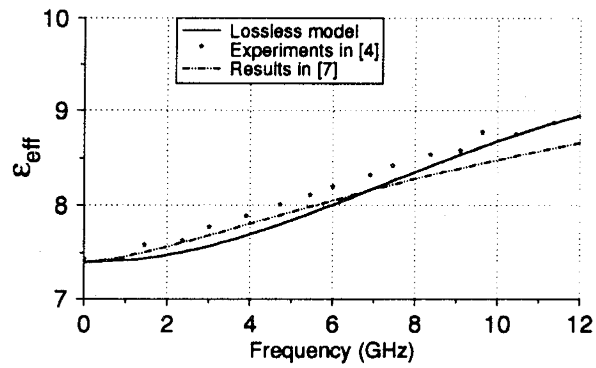

Fig. 6. Effective dielectric constant versus frequency for an alumina substrate with $\epsilon_r = 10.1$ and w/h=2.5.

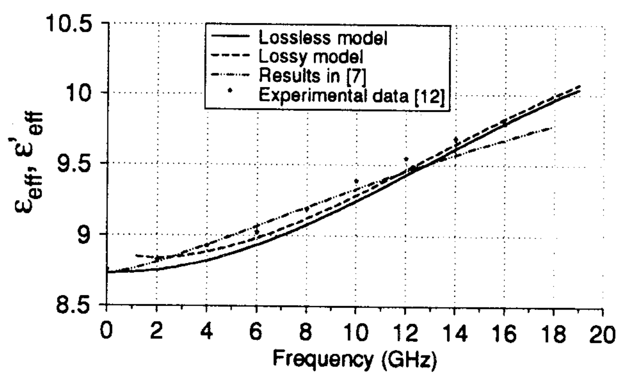

Fig. 9. Effective dielectric constant versus frequency for a sapphire substrate with $\epsilon_r = 11.39$ and w/h=3.75.

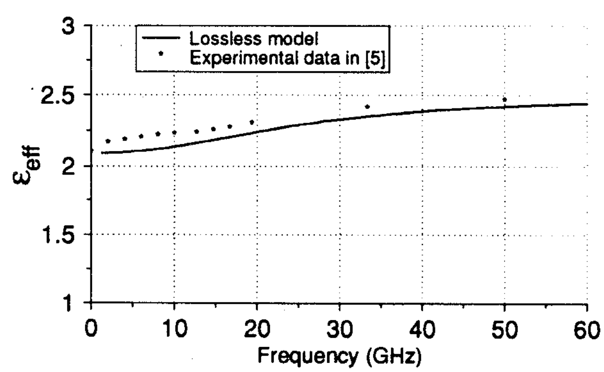

Fig. 7. Effective dielectric constant versus frequency for a fluorglas substrate with $\epsilon_r = 2.5$ and w/h=3.04.

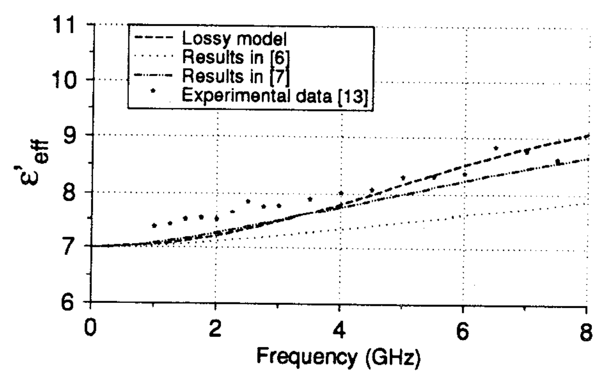

Fig. 10. Effective dielectric constant versus frequency for a RT/duroid 6010 substrate with $\epsilon_r = 10.5$ and w/h=0.88.